

Craze growth and craze interactions

N. J. MILLS

Department of Physical Metallurgy, University of Birmingham, Birmingham, UK

A dislocation array method of two-dimensional stress analysis has been used to interpret commonly-observed features of craze growth in glassy polymers. It is deduced from the lack of deviation of craze pairs that the tensile stress across an air craze is more than 90% of the applied stress. Craze interactions occur at an early stage of growth when their penetration normal to the free surface exceeds 20% of their separation in the direction of the applied stress. It is suggested that the growth kinetics of crazes are controlled by the potential energy changes in the surrounding elastic material, which in turn are affected by the geometric interactions of crazes.

1. Introduction

Three recent reviews [1-3] of crazing in glassy plastics, give the background to the problem of the stress analysis of craze growth. The stress analysis of isolated crazes has been based either on particular models such as that of Dugdale [4] for a line plastic zone at the end of a crack, or on Fourier transform analysis of the observed craze thickness profiles [3]. The former approach has been used, for the crazes that occur at crack tips in polymethyl methacrylate (PMMA) [5, 6], polyvinylchloride (PVC) [7] and polycarbonate (PC) [8] to analyse both the length, and the thickness profile. The latter approach was adapted to crazing by Knight [9] and then applied to the thickness profiles of crazes observed in PMMA by holography [10] and in polystyrene by electron microscopy [11]. In all of the above works the thickness profile of a craze has been observed on a particular x - y section (Fig. 1) and experimental conditions have been such (through-thickness crack etc.) that the craze has the same profile at all other x - y sections. However, Fig. 1 indicates that general crazes that initiate from a free surface do not have a constant cross section parallel to the surface plane, and that other crazes may be sufficiently close that the possibility of the stress fields of crazes interacting cannot be ignored.

The purpose of trying to analyse the stress fields of crazes is to be able to make predictions of the growth in size and change of shape of crazes prior to crack initiation. As the crack will

probably initiate from the largest craze it is important to know over what range of stresses crazes will appear, and what size distribution of crazes is likely. Reports of craze growth have been of the surface length variation only, (Fig. 1). At ambient temperatures, craze lengths in polycarbonate have been found to be proportional to log time [12], and craze lengths in polystyrene have been found to be proportional to time [13]. The shape of crazes in PMMA has been observed to change with time [14], as a result of mutual

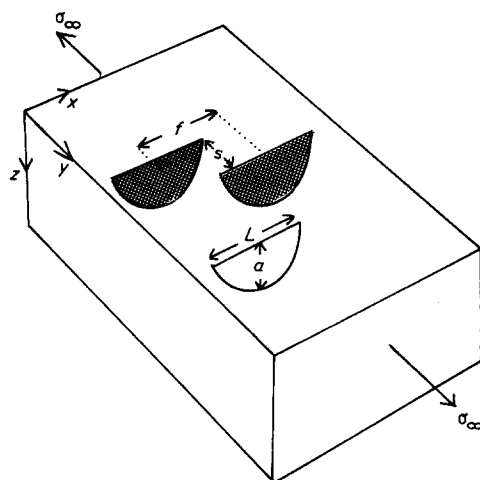


Figure 1 Three-dimensional sketch showing a few surface crazes and the dimensional parameters used to characterize their growth. L is the surface length, a is the penetration, s is the separation, and f is the off-set of a pair of crazes.

hinderance. However, it is not known whether this is a general phenomenon.

The new stress analysis method described in a companion paper [15] is applied here in order to determine if craze interaction is a significant factor influencing craze growth. Before this analysis is undertaken the major existing theories of the micromechanics of craze growth are reviewed in order to identify their strengths and weaknesses.

2. Existing models of craze micromechanics

2.1. The Verheulpen-Heymans–Bauwens model

This two-dimensional analysis [12] concerns a central craze in an infinite sheet, formed perpendicular to a tensile stress, σ_∞ , applied at infinity (Fig. 2a). It assumes that the normal traction across the craze differs in value in the craze body, σ_c , and craze tip, σ_e , regions. In it the craze is conformally mapped from the z -plane into a ζ -plane, in which it becomes a unit circle, and the stress components are given in terms of the co-ordinate ζ . On checking this analysis an error or misprint was found: in Equation A24, for σ_x , the sign before Im should be a plus and not a minus. However, for the expressions A24 to A27 in [12] the stress field can be expressed more concisely in terms of the co-ordinate $z = x + iy$

$$\frac{1}{2}(\sigma_{xx} - \sigma_{yy} + 2i\sigma_{xy}) = \frac{\sigma_\infty}{2} + \frac{(\sigma_e - \sigma_c)iy a^2 \sin 2\beta}{\pi(z^2 - a^2)^{\frac{1}{2}}(z^2 - a^2 \cos^2 \beta)^{\frac{1}{2}}} \quad (1)$$

and

$$\frac{1}{2}(\sigma_{xx} + \sigma_{yy}) = \frac{\sigma_\infty}{2} + \frac{\sigma_e - \sigma_c}{\pi/2} \left[\text{Re} \tan^{-1} \left(\frac{z \tan \beta}{(z^2 - a^2)^{\frac{1}{2}}} \right) - \beta \right], \quad (2)$$

where

$$\beta = \frac{\pi}{2} \left(\frac{\sigma_\infty - \sigma_c}{\sigma_e - \sigma_c} \right) = \cos^{-1} \left(1 - \frac{R}{a} \right),$$

R is the length of the craze tip region and a is the total craze half-length. These equations could also have been derived from the Westergaard stress function [16] given for line yielded zones ahead of a central crack, with a stress at infinity of $\sigma_\infty - \sigma_c$, and a yield stress, $\sigma_e - \sigma_c$ and adding a uniform stress field such that $\sigma_{yy} = \sigma_c$, $\sigma_{xy} = \sigma_{xx} = 0$ everywhere.

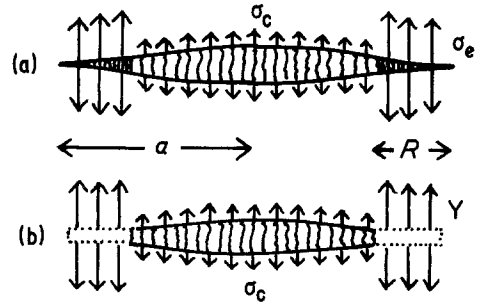


Figure 2 Two-dimensional models of crazes proposed by (a) Verheulpen-Heymans–Bauwens and (b) Argon and Salama. The craze opening profile is magnified greatly in the y -direction, and the level of local tensile stresses is indicated by the lengths of the arrows.

The physical assumptions in the model are open to a number of criticisms:

(i) The analysis is two-dimensional, whereas real crazes have a three-dimensional form and they usually grow away from a free surface (Fig. 1).

(ii) A single craze is considered, whereas usually a number of crazes grow simultaneously; hence any craze interactions are neglected.

(iii) The craze initiation problem is not considered; a craze is assumed to pre-exist, with both a body and a tip region. There is considerable evidence that crazes initiate from surface grooves [17].

(iv) Equation 2 implies that $\sigma_c < \sigma_\infty$ and $\sigma_e > \sigma_\infty$, and thus that crazes should only appear when σ_∞ falls between these two limits. If, as in (iii), surface grooves were present then the restriction $\sigma_\infty > \sigma_c$ would no longer be necessary. Some observations of the craze shape at crack tips in PMMA and PC have been analysed in terms of a constant σ_c that is approximately equal to the uniaxial tensile yield stress [7, 8], in which case σ_c would be larger than σ_∞ .

(v) Any purely elastic analysis is unable to predict craze growth kinetics. In the original version of the model it was assumed that no material was drawn into the craze through its sides, hence, the material in the craze extends by creep. An empirical visoelastic model was then used to describe the creep process in the craze and to predict the variation of craze length with time, t , which was found to be of the form

$$a = A \ln(t/t^*), \quad (3)$$

where A and t^* are constants. In a more recent paper [18] it is concluded that craze thickening is due to drawing in new material through the

craze sides. It is still observed that the crazes grow with a constant ratio of maximum craze opening, δ_0 , to length, a . Since the elastic analysis gives, for $R \ll a$,

$$\delta_0/a = (1 - 2\nu)(\sigma_\infty - \sigma_c)/2G \quad (4)$$

where ν is Poisson's ratio, and G is the shear modulus, then, as the applied stress σ_∞ is constant, σ_c is also constant. It is postulated that craze growth kinetics are similar to those of a neck growing in a tensile specimen under creep loading, i.e. that the rate of drawing in material through the craze walls entirely controls the craze length, and there are no equilibrium values of the craze length.

(vi) It follows from (v) that the craze tip region, of length R , will grow longer as the craze length increases, if the craze tip stress σ_c remains constant. This is physically unrealistic for a craze initiation process of the type proposed by Argon, which would be more likely to be a region of constant length.

2.2. The Argon–Salama model

This model [13] differs from the Verheulpen–Heymans–Bauwens model in that it assumes a single tensile stress level, σ_c , normal to the craze, and a conventional (non-craze) yielded zone with stress, Y , ahead of the craze tip, see Fig. 2b. Otherwise the mathematics of the stress analysis are identical. It is assumed that the yielded zone length, R , is much smaller than the total craze length a , so, in these circumstances of “small scale yielding” Equation 2 reduces to

$$\left(\frac{\sigma_\infty - \sigma_c}{Y - \sigma_c}\right)^2 = \frac{R}{a}. \quad (5)$$

It is assumed that both the yielding and the crazing processes are strainrate and temperature sensitive, and that this sensitivity takes a form, developed by Argon [19] in which the plastic strain rate $\dot{\epsilon}$ is related to the yield stress, Y , and the temperature, T , by

$$\dot{\epsilon} = \dot{\epsilon}_0 \exp \left[-\frac{A}{T} \left(1 - \frac{Y}{\hat{Y}} \right)^{\frac{5}{2}} \right], \quad (6)$$

where A , $\dot{\epsilon}_0$ and \hat{Y} are constants.

It is assumed that the length, R , of the yielded region remains constant as the craze grows; hence, according to Equation 5, the value of the craze stress σ_c must increase to a value almost equal to

the applied stress, σ_∞ . The craze growth velocity, v , is directly proportional to the strain rate in the yielded zone, so, after an initial transitional period, the craze velocity settles down to a steady value proportional to the right-hand-side of Equation 6.

The Argon–Salama model is similar in many ways to the Verheulpen–Heymans–Bauwens model, and it has many of the same limitations. The craze growth kinetics are determined by the yielding kinetics of the bulk polymer: the Argon model states that a constant imposed stress produces a constant strain rate whereas the Verheulpen–Heymans–Bauwens model states that in the neck propagation process under creep conditions the neck velocity is inversely proportional to the elapsed time. Both of the models work for the limited sets of data to which they are applied, but they are incompatible, and neither are able to predict the effect of craze interactions.

3. The use of dislocation array stress analysis methods to analyse craze growth

3.1. Introduction

The details of the model and the checks on its performance have been described in the companion paper [15]. It enables more complex geometrical arrangements of crazes than those in Fig. 2 to be analysed. At this stage the complexities of the kinetics of the yielding and void formation processes involved in crazing are not brought into the analysis, nor is the non-linear viscoelastic behaviour of polymers. The craze is initially modelled as a plastic region in a linearly elastic material (Fig. 3a) and the potential energy, V , of the elastic region, that is left when the craze is cut out, is calculated. Depending on the situation being modelled, the driving force for craze growth is different. In Fig. 3a and b where there is no crack present, when a uniform stress field equal to the applied stress at infinity σ_∞ is subtracted, the remaining problem is that of an internally pressurized crack, either with (a) or without (b) a yielded zone at the end. The driving force for the growth of this “crack” is the potential energy release rate, $-dV/da$. For most geometrical arrangements, this quantity is always positive so the “crack” never stops growing. When the uniform stress field of σ_∞ is replaced, the craze will never stop growing either, though the kinetics of its growth may depend on the rate at which plastic work, P , is required to thicken the craze.

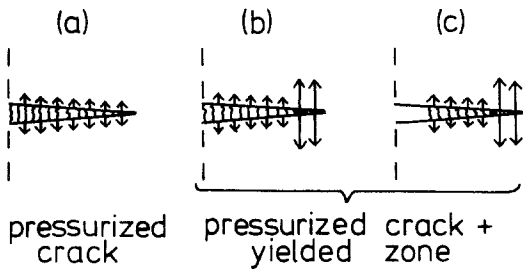


Figure 3 Two-dimensional models of crazes discussed in this paper. The vertical line is either a free surface or the centre line of an infinite sheet. (a) single level craze stress model, (b) two-level craze stress model and (c) crack plus two-level craze stress. Below each is the description of the actual problem solved by stress analysis, when the stress field at infinity is removed.

In Fig. 3c where the craze has grown from a pre-existing surface or central crack, it is assumed that the crack length, a , remains constant. It was shown in [15] that the line yielded zone of length, R , (in this case the craze) would grow if $-dV/dR > 0$, and that this gave identical results to those found using other criteria for yielded zone growth. It is possible in this situation that the craze reaches an equilibrium length.

3.2. Predictions of the growth of pairs of crazes in the surface plane

It is commonly observed that crazes occur in close proximity on a free surface, and that they remain parallel to one another. The simple explanation would be that they are all growing normal to the applied tensile stress, but this neglects the local stress fields around each craze.

The situation modelled was an isolated pair of crazes that are initially parallel but off-set, see Fig. 4. Referring back to the three-dimensional sketch in Fig. 1, this two-dimensional model could apply approximately near the free surface when the craze penetration is greater than the craze surface lengths.

The initial calculations were for a single level of craze stress, σ_c , that was constant along the craze; accordingly, σ_c must be less than the applied stress, σ_∞ , in order for the craze growth to be energetically possible. Fig. 4 shows the variation in the predicted growth paths of the inner craze tips of the pair, as the ratio σ_c/σ_∞ is varied, with values of the quantity $-dV/da$, given in Table I.

It can be seen that as σ_c/σ_∞ increases towards 1.0 the deviation of the growing crazes decreases, but the values of $-dV/da$ also decrease sharply.

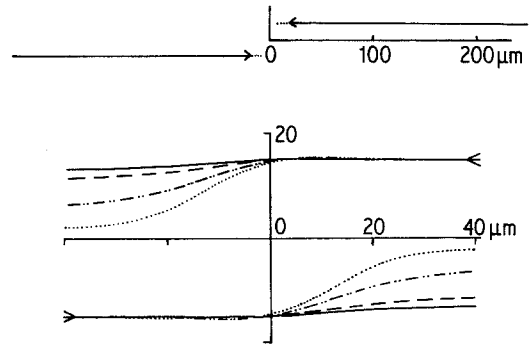


Figure 4 Predictions for the growth paths of two off-set crazes; the small inset shows the complete geometrical situation, whereas the larger part shows the paths of the crazes relative to the centre of symmetry, 0 for single craze stress levels of 20% of σ_∞ , 70% of σ_∞ ----, 90% of σ_∞ - - - - and 95% of σ_∞ ———.

For $\sigma_c/\sigma_\infty \geq 1$ craze growth is only possible if there is a local stress raiser, for example a crack, present. The other extreme of $\sigma_c = 0$, which represents two cracks growing has already been analysed [15]; in this case the cracks deviate very strongly in growth direction once they overlap. In order to predict whether craze growth is preferred at the inner or at the outer pair of craze tips, the values of $-dV/da$ for craze growth at the two ends can be compared. Initially, the level of $-dV/da$ is higher at the inner craze tips, and it remains so until the crazes have overlapped by 1.4 times the craze separation, whereupon the value at the outer craze tips becomes larger.

When the surface crazes on an unplasticized PVC tensile specimen were examined (Fig. 5), many pairs of overlapping crazes were found, but there was no detectable deviation in the craze growth direction. This seems to be true for the many types of glassy polymers examined, and for nearly all the published photographs of surface crazes [1, 2]. However, when thin films ($\sim 1 \mu\text{m}$ thick) are stressed in tension [20] deviations of the type shown in Fig. 4 are observed when the craze-tip separation, s , is small. Therefore the single craze stress model can only match experimental observations if $0.9 < \sigma_c/\sigma_\infty < 1.0$. Deviations are sometimes observed with solvent crazed polymers; Fig. 6 shows the surface of polycarbonate, crazed in the presence of 25 vol% toluene 75 vol% propanol mixture [21]. This produces large weak crazes that deviate in the way indicated by Fig. 4.

Recent work on polystyrene crazes by Kramer [3] has suggested that a two-level craze stress

TABLE I Predicted potential energy release rates (Jm^{-2}) for the growth of the inner ends of a pair of off-set crazes, initially $230 \mu\text{m}$ long, $\sigma_\infty = 50 \text{ MN m}^{-2}$, $E = 3 \text{ GN m}^{-2}$, $\nu = 0.45$

Craze growth (μm)	$\sigma_c/\sigma_\infty(\sigma_e/\sigma_\infty)$				
	0.2 (—)	0.7 (—)	0.9 (—)	0.9 (1.2*)	0.95 (—)
12	405	58.2	6.48	5.99	1.62
24	581	80.0	8.62	8.54	2.12
36	409	48.9	4.89	5.63	1.18
48	143	19.1	2.16	2.72	0.54
60	47	8.2	1.09	1.42	0.29

*A craze tip region $6 \mu\text{m}$ long.

model produces a more accurate representation than the single level craze stress model. It is possible for the craze tip stress, σ_e , to be greater than the applied stress, σ_∞ , and the craze still to grow, provided that the craze body stress, σ_c , is less than σ_∞ . Since details of the size of the craze tip region are only known for polystyrene, similar values have been assumed to apply for PVC. For a tip region $6 \mu\text{m}$ in length, with $\sigma_e = 1.2 \sigma_\infty$ and with $\sigma_c = 0.9 \sigma_\infty$, the change in the craze growth path predicted by the two-level craze stress model is negligible compared with that predicted by the single craze stress model.

It can therefore be deduced from photographs such as Fig. 5 that:

(i) The tensile stress transmitted across the craze is a high percentage of the applied stress, probably $> 90\%$;

(ii) Crazes only grow past each other to a limited extent because craze interactions reduce the energetic driving force at the inner pair of craze tips.

3.3. The energetics of craze growth normal to a free surface

Craze growth normal to a free surface is ultimately more important than growth along the surface

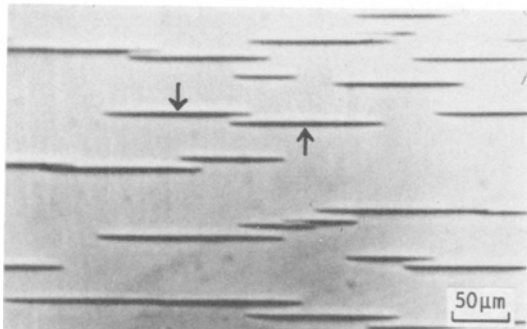


Figure 5 Surface crazes in unplasticized PVC; the arrows show the initiation points of two crazes.

because the direction of the eventual crack growth (if any) will be normal to the surface. It should not be assumed that the craze surface length, and the degree of penetration are equal.

In the two-dimensional model of craze growth from a free surface a semi-infinite sheet $y \geq 0$ is subjected to a single tensile stress component σ_∞ at infinity that acts in the x -direction. Since the craze(s) grow in the plane perpendicular to the largest principal tensile stress they will grow in the y -direction; consequently only the energetics of craze growth need be investigated. The results can be related to the three-dimensional situation shown in Fig. 1 for the case when the craze surface length is much greater than the degree of penetration.

The simplest situation to analyse is that of a single edge craze that has a constant tensile stress σ_c across it (Fig. 3a). If a uniform stress $\sigma_{xx} = \sigma_\infty$ is applied everywhere the resulting problem is that of an edge crack under a uniform internal pressure, p , of $\sigma_\infty - \sigma_c$ in a sheet that is stress-free at infinity; the potential energy, V , of this system is given by [22]

$$V = -0.9879(1 - \nu)p^2a^2/G. \quad (7)$$

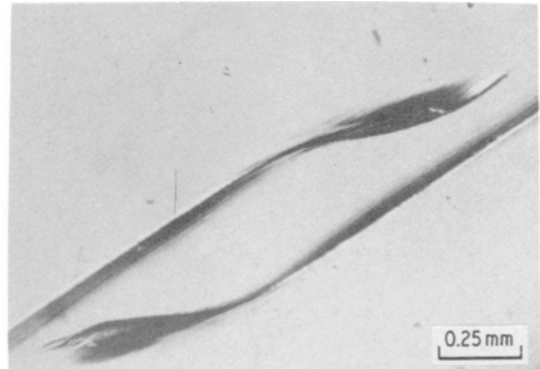


Figure 6 Surface crazes in polycarbonate in a 1:3 v/v toluene to propanol mixture (Crown copyright).

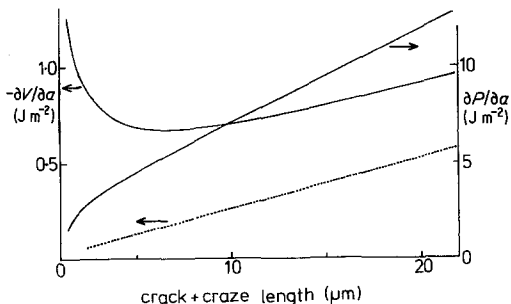


Figure 7 Variation of the potential energy release rate, $-dV/da$, and of the rate of dissipating plastic energy in the craze, dP/da , for an edge crack $1 \mu\text{m}$ in depth with a two-level craze stress. The craze tip, $0.5 \mu\text{m}$ in length, has a stress of 60 MN m^{-2} , the craze body has a stress of 45 MN m^{-2} and the applied σ_∞ is 50 MN m^{-2} . The material used was of Young's modulus 3 GN m^{-2} and Poisson's ratio 0.45 . For comparison the dotted line shows the $-dV/da$ relation without the crack being present.

The value of $-dV/da$ will increase linearly from zero when the craze has zero length. This model leads to unrealistic predictions of craze growth: firstly, there is no energetic driving force for craze initiation, and secondly, the craze growth velocity, v , should accelerate as the craze length increases, if it is assumed that v is a monotonically increasing function of $-dV/da$ such that

$$V = f(-dV/da), \quad (8)$$

where f is a monotonically increasing function. The assumption of behaviour in keeping with Equation 8 is a reasonable one, considering the widely-found relationship of the same form for the crack velocity of polymers (see later).

The first modification of the simple model is to supply a surface groove that will help initiate the craze (Fig. 3b). Although a two-stress craze model is used it does not modify the predictions markedly; Fig. 7 shows the variation in $-dV/da$ with craze length. The effect of the surface crack (of length $1 \mu\text{m}$) is to provide a non-zero initial value of $-dV/da$; then after a growth of about $10 \mu\text{m}$ $-dV/da$ increases proportionally with craze length. (The major part of this increase is supplied by the crack opening.) This modification has now overcome the problem of craze initiation but still predicts accelerating craze growth. It should be noted that the absolute level of $-dV/da$ depends on the assumed values of σ_c , σ_∞ and the surface crack length. Fig. 7 also shows the rate at which plastic work is done, dP/da , as

the craze grows; this is roughly ten times larger than $-dV/da$ and it too tends to increase linearly once the initial stages of craze growth are complete. Hence, since it is impossible for the potential energy release to supply the whole of the plastic work done in the craze, the majority of the plastic work must be supplied by the loading system.

The second modification of the simple model is to consider the simultaneous growth of a number of crazes from a free surface. If the crazes are initiated without the aid of surface cracks, then Fig. 8 shows that the value of $-dV/da$ starts from zero, but reaches a plateau value of $(\sigma_\infty - \sigma_c)^2(1 - \nu^2)s/2E$, where s is the craze separation, when the craze length exceeds 30% of the craze separation (this was the problem analysed in the previous paper [15]). The effect of including small surface cracks in the model is dramatic; if the cracks are $1 \mu\text{m}$ in depth (a probable value for the surfaces of most glassy polymers) and separated by $200 \mu\text{m}$, then the value of $-dV/da$ is nearly independent of the craze length, whereas if the cracks are $2 \mu\text{m}$ in depth and $200 \mu\text{m}$ apart the initial value of $-dV/da$ would, for the stresses and elastic constants used, be of the order of three times the asymptotic value. It can be deduced from Fig. 8 that, if the polymer surface contains a distribution of scratches of different depths, then the deepest scratches will initiate crazes

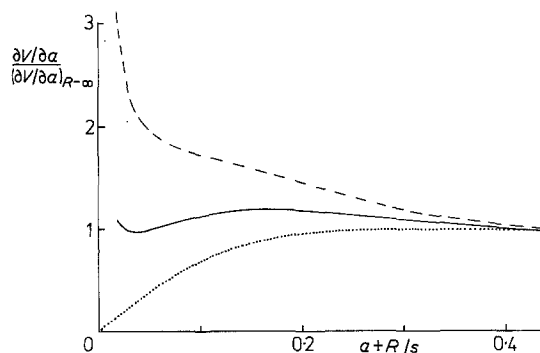


Figure 8 Craze growth from a series of equal parallel edge cracks with the same stress parameters and elastic constants as Fig. 7. — for a crack length/crack separation ratio of $1:200$ ($0.5 \mu\text{m}$ crack separated by $100 \mu\text{m}$) and --- for a crack length/crack separation ratio of $1:100$. $-dV/da$ is normalized by dividing it by the asymptotic value when $R \rightarrow \infty$, and the craze length R is normalized by dividing by the separation. The dotted curve is for a single level craze stress model growing from a free surface without a crack.

first, and these crazes will grow rapidly until they begin to interact strongly when their length/separation ratio exceeds 0.3.

The conclusions of this section are that:

(i) Any realistic model of craze initiation must include surface scratches or grooves;

(ii) craze interactions in this two-dimensional model lead to a constant energetic driving force $-dV/dR$ when the craze lengths exceed 30% of their craze separations.

4. Comparison of predictions with experimental data on craze growth

A variety of different kinetic laws have been used to describe the experimentally-determined craze growth data for polymers. The data available seems to be exclusively measurements of the surface length, L , (Fig. 1) rather than of the penetration from the free surface of the craze. Thus, at a temperature of 78 K, where shear flow is minimized, crazes grow at a constant velocity [23] in several polymers; at room temperature [12] and at 283 K [24] crazes in polycarbonate grow with lengths proportional to log time. For higher stresses, in the results of [24], there is evidence of growth ceasing after a period of time. In [13] where "the lengths of only isolated crazes were measured as a function of time" constant craze-tip velocities were found for polystyrene and polymethylmethacrylate (PMMA). However, in [14] the craze-tip velocity was found to decrease with time for PMMA at 23°C at a stress of 25 MN m^{-2} , when only isolated crazes grew, and to decrease to zero after about 100 min for a stress of 40 MN m^{-2} , due to craze interactions. Results obtained in this laboratory [25] on polyvinylchloride confirm that craze growth ceases after approximately 200 minutes when high stresses are applied (Fig. 9). Prior to this the surface length increase is proportional to log time for times between 50 and 150 min.

It may well be that in a number of investigations the form of the craze length against time relationship would have changed had the experiment been continued for longer. When the PVC specimens were sectioned in a plane containing the tensile stress direction and the surface normal, it was apparent that the crazes were fairly regularly

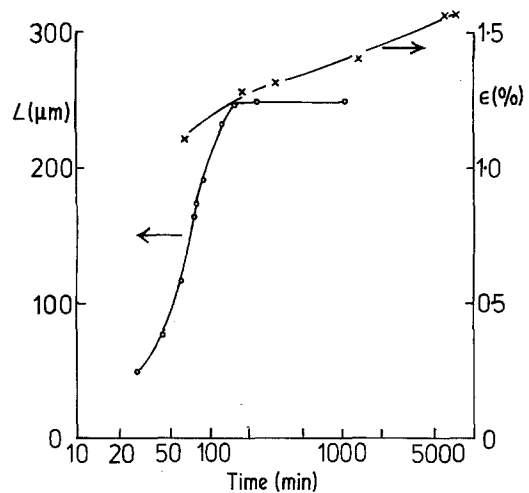


Figure 9 Variation of craze surface length, L , with time for PVC at 20°C under an applied $\sigma_{\infty} = 35 \text{ MN m}^{-2}$. The variation of the creep strain with time is also shown.

spaced and had nearly uniform lengths (Fig. 10). There was no evidence of localized yielding in the form of shear bands. Hence, there is a strong probability that the elastic stress fields of the crazes have interacted in a way similar to that modelled in Section 3.3. The effects of this elastic stress field interaction can be visualized using photoelastic models [26], for example in the case of parallel edge cracks, see Fig. 11. It can be seen that there are severe stress gradients near and between the crack tips, but, at only a short distance behind the crack tips the stress has decayed to a very low value. Hence, a surface layer of depth just less than the crack lengths is stress relieved. The two-dimensional stress analysis of Section 3.3 was able to predict maximum shear stress contour maps identical to that in Fig. 11, providing a useful check on the validity of the analysis.

Further evidence of the interaction of growing crazes was obtained during work on fatigue crack initiation in polycarbonate [27]. These tests were made on four-point bend specimens, so that it was possible on have, on a single specimen after a given number of fatigue cycles, a range of maximum surface strains in the region between the inner and outer load points. It was found that the density, ρ , of "mature" crazes (crazes mm^{-2}) increased as the maximum strain increased and their length, L , decreased. If the area of surface that is stress

*The crazes are mature in the sense that their growth in surface length has been stopped by the formation of shear bands at both ends.

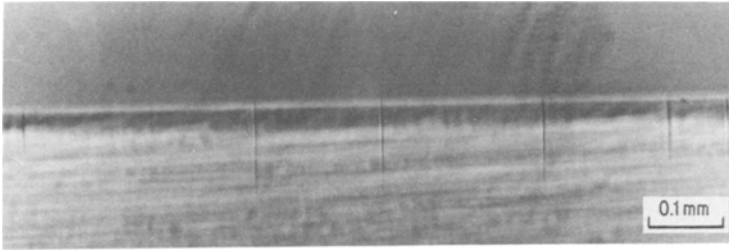


Figure 10 Crazes in PVC seen normal to the free surface, so that the penetration depth, a , is visible.

relieved by a craze is proportional to L^2 then it might be expected that ρL^2 will be a constant provided that there are sufficient crazes present to stress relieve the whole surface. Using the data from Fig. 1 of [27] it was found that ρL^2 had a mean value of 0.26 with a standard deviation of 0.07 for values of ρ between 43 and 189 crazes mm^{-2} .

5. Discussion

First the question of whether craze growth is a direct consequence (or cause) of some other process will be discussed. It has been suggested that craze growth is directly linked to the creep process, initially as a result of the creep of the craze material [12], and more recently because the drawing-in of material through the craze sides is a necking process that occurs under creep conditions [18]. If this were true then it would be expected that the kinetics of craze growth, and of the other process, would be identical. However it is clear that Fig. 9, of craze growth in PVC, is quite unlike

the creep strain variation in the same material, which is initiated earlier, and shows no sign of reaching an equilibrium value after 200 min. In addition, the craze growth cannot be linked to neck propagation for this material, since it was experimentally impossible to get necking to occur under creep conditions, brittle fracture always occurring first. While one negative example does not rule out the possibility that craze growth is linked to other processes in some polymers under some conditions, it does show that this is not universally true.

Next, the conclusion of a recent paper [10] that “craze growth criteria based on the concept of a constant critical total strain energy release rate cannot be correct” will be examined, since, if it were true, it would nullify the usefulness of the analysis in this paper. The authors calculated by three methods the strain energy increments ΔU that occur when a craze grows in PMMA in a methanol environment under fixed grip conditions. Their first method of calculating ΔU was

$$\Delta U^{(1)} = -g\Delta P/2B, \quad (9)$$

where g is the fixed grip displacement, ΔP is the change in the applied tensile load, and B is the specimen thickness. However, in fact the elastic material is not under “fixed grip” conditions: although the grips at the ends of the specimen are fixed, the craze surfaces change their displacements and these are as much a part of the boundary of the elastic region as are the grips. Their third method of calculating ΔU was

$$\Delta U^{(3)} = \int_0^a \overline{\sigma_y}(z) 2W(z) dz, \quad (10)$$

where a is the craze length, $\overline{\sigma_y}$ is the mean tensile stress normal to the craze before and after the craze growth, $2W$ is the total craze opening displacement (ΔW might be more appropriate) and z is the co-ordinate axis parallel to the craze. This quantity is not the total strain energy incre-

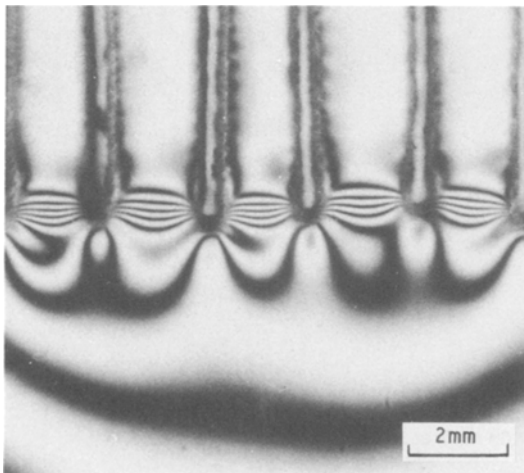


Figure 11 A photoelastic model of parallel edge cracks, with the isochromatic (maximum shear stress) contours visible, under a tensile stress at infinity normal to the cracks.

ment either; it is in fact the increment of plastic work in the craze, and as such it should be subtracted from $\Delta U^{(1)}$ to find the real change in the stored elastic energy. Consequently the strain energy release rate G can be calculated using

$$G = (\Delta U^{(1)} - \Delta U^{(3)})/\Delta a, \quad (11)$$

where Δa is the craze growth increment, yielding G -values that vary erratically between -60 and $+20 \text{ J m}^{-2}$. This contrasts with their calculations which purport to show that G rises rapidly from 80 J m^{-2} to a value of 600 J m^{-2} when the craze stops growing. Consequently, the authors' data does not rule out craze growth criteria based on G , it merely shows that G is difficult to measure experimentally and that it is probably smaller than 50 J m^{-2} for the polymeric system investigated.

It was proposed in [15] that the equilibrium length, R , of a line yielded zone occurred when the potential energy, V , of the elastic material had a minimum value, i.e. when $-dV/dR = 0$. The shape of a craze approximates well to a line yielded zone because its length-to-thickness ratio is very large. This criterion can only be used if the applied stress does not fall; if the criterion is applied to crazes this must be the case because crazes do not heal up when the load is reduced or removed. If the criterion is applied to crazes, then $-dV/dR$ is the energetic driving force for the craze length R to increase, in the same way that, in fracture mechanics, $-dV/da$ is the energetic driving force for the crack length, a , to increase.

In the original version of linear elastic fracture mechanics there was no possibility of slow crack growth. The crack grew if, and only if, the stress intensity factor, K , at the crack tip exceeded the material parameter, K_c . This criterion has subsequently been empirically modified so that, for most materials, the crack velocity is a function $V(K)$, and that usually the crack will not grow at a detectable rate if K does not exceed a threshold value. The analysis presented so far in this paper has not included the kinetics of craze growth, but it would seem reasonable that the craze growth velocity was a function of some controlling parameter and, hence, the suggestion of Equation 8. It is also probable that a craze will not grow at all if the value of $-dV/dR$ does not exceed a threshold value that is a material parameter. Further experimental work is needed in order to establish the validity of Equation 8, and if it is, to establish the nature of the function f .

Next, the predictions of the energetics analysis in this paper are compared with observations of craze growth. In modelling the craze it is possible to use a one-level craze stress model, a two-level craze stress model, or some more complex variation. It is clear that the single-level craze stress model leads to unrealistic predictions of the equilibrium length of crazes; if the craze stress σ_c is less than the applied stress σ_∞ then the craze lengths should grow to an infinite size, in spite of any craze-craze interactions; if $\sigma_c > \sigma_\infty$ then the craze stress can only be very slightly larger than σ_∞ otherwise the craze equilibrium lengths will be impossibly small, even allowing for large surface grooves; however, it is observed that crazes grow for a significant range of σ_∞ values. The two-level craze stress model surmounts these difficulties provided that the stress at the end of the craze, σ_e , is greater than the applied stress, σ_∞ , and provided that the stress in the body of the craze is slightly less than σ_∞ .

To proceed further with the discussion of craze growth, the two dimensions, of surface length and of penetration from the free surface, will be treated separately so that comparison with two-dimensional models can be more easily made.

Firstly, it is clear from the discussion of the results shown in Table I that the close proximity of the innermost craze tips of a pair of crazes leads to a reduction in $-dV/da$ at these craze tips once they overlap. However, it does not decrease the corresponding quantity at the outer craze tips. Hence, it is possible to understand why pairs of partially overlapped crazes are often observed, but it does not immediately explain why the total length of the craze pair does not continue to increase. However, when the penetration from the surface of a number of equally spaced crazes is analysed (Fig. 8) it is clear that, once the craze length/craze separation ratio, a/s becomes greater than 0.3, the potential energy release rate $-dV/da$ reaches a plateau level; the consequence of this is that the tensile stress near the free surface is reduced to σ_c , the tensile stress across the craze body. If the craze surface length, L , is now reconsidered in the light of this "stress relieved" surface layer it is clear that there is no longer any driving force for growth of the craze surface length. Therefore, the curves delineating the craze tip are effectively pinned at the free surface at this stage, and further growth can only occur by increased penetration.

Secondly, although the analysis made of the energetics of the increase in craze penetration from the free surface is incomplete, it demonstrates several interesting features. It predicts that craze initiation is only possible if there are surface grooves; otherwise $-dV/da$ is zero when the craze length is zero. For the two-dimensional model, shown in Fig. 9, $-dV/da$ reaches a steady value per craze once each craze in the array has grown longer than about 30% of the craze separation. This conclusion would not be modified if a more realistic three-dimensional model were analysed with flat finger-like crazes all advancing at the same rate, (like Fig. 1 but with a greater craze penetration), since each craze would liberate potential energy from the prism of material surrounding it rather than from a plate of material. However as the craze-tip curve-length increases as the craze grows there would be an additional expenditure of energy compared with the two-dimensional model.

These ideas are supported by the analysis of the experimental observations of craze growth in polycarbonate: each mature craze would appear to occupy a surface area of roughly $4L^2$.

If the postulate is made that craze growth will not occur unless

$$-dV/da \geq C, \quad (12)$$

where C is a material constant, then it is possible to explain why craze growth could stop even when the applied stress σ_∞ exceeds the craze body stress σ_c : if a number of crazes of equal length and separation develop, then the value of $-dV/da$ will depend on s , and not on the craze length, if $a > 0.3s$; consequently, it is possible that Condition 12 is not met, and so craze growth stops. This stable situation could break down if one of the crazes becomes larger than the rest.

It is evident that further research must be done on the micromechanics of crazing. Kramer [3] has discussed whether it is the micromechanics of the craze tip or of the processes occurring at the sides of the crazes that control craze growth kinetics. Whichever it is, it seems possible that it is the potential energy release rate of the elastic material that is the driving force. There seems little point in refining any models of craze growth kinetics [12, 13] until it becomes apparent what are the craze micromechanisms, and what are the geometric factors in each experiment (size of surface grooves, relative positions of crazes etc.).

6. Conclusions

The following conclusions can be drawn from this investigation:

(a) On the basis of observations that crazes rarely deviate in direction when they grow past each other, the tensile stress across the body of an air craze must be at least 90% of the applied tensile stress on the specimen.

(b) Craze interactions are very important in limiting the surface growth of crazes. They also alter the energetics of craze penetration from the free surface, largely as a result of the presence of neighbouring crazes in the line of the applied tensile stress.

(c) The principle of minimizing the potential energy of the elastic material surrounding the craze seems to be a useful one for predicting the equilibrium length of crazes. It is possible that the differential $-dV/da$ controls the craze velocity.

Acknowledgements

I am grateful to Ms J. Harris of the Procurement Executive, Ministry of Defence for providing the photograph used as Fig. 6, and to the Computer Centre of the University of Birmingham for providing computing facilities.

References

1. R. P. KAMBOUR, *J. Polymer. Sci. Macromol. Rev.* 7 (1973) 1.
2. S. RABINOWITZ and P. BEARDMORE, "CRC Critical Reviews in Macromolecular Science", Vol. 1 (CRC Press, Cleveland, Ohio, 1972).
3. E. J. KRAMER, in "Developments in Polymer Fracture" edited by E. H. Andrews (Applied Science Publishers, London, 1979).
4. D. S. DUGDALE, *J. Mech. Phys. Sol.* 8 (1960) 100.
5. G. P. MARSHALL, L. E. CULVER and J. G. WILLIAMS, *Proc. Roy. Soc.* A319 (1970) 165.
6. H. R. BROWN and I. M. WARD, *Polymer* 14 (1973) 469.
7. N. J. MILLS and N. WALKER, *ibid.* 17 (1976) 335.
8. R. A. W. FRAZER and I. M. WARD, *ibid.* 19 (1978) 220.
9. A. C. KNIGHT, *J. Polymer. Sci.* A3 (1965) 1845.
10. E. J. KRAMER, H. G. KRENZ and D. G. AST, *J. Mater. Sci.* 13 (1978) 1093.
11. B. D. LAUTERWASSER and E. J. KRAMER, *Phil. Mag. A* 39 (1979) 469.
12. N. VERHEULPEN-HEYMANS and J. C. BAUWENS, *J. Mater. Sci.* 11 (1976) 1.
13. A. S. ARGON and M. M. SALAMA, *Phil. Mag.* 36 (1977) 1217.
14. J. OPFERMANN and G. MENGES, Fourth International Conference on Fracture, University of Waterloo, 1977 (Pergamon, New York, 1978) p. 1095.

15. N. J. MILLS, *J. Mater. Sci.* **16** (1981) 1317.
16. F. M. BURDEKIN and D. E. W. STONE, *J. Strain Analysis* **1** (1966) 145.
17. A. S. ARGON and J. G. HANNOOSH, *Phil. Mag.* **36** (1977) 1195.
18. N. VERHEULPEN-HEYMANS, *Polymer* **20** (1979) 356.
19. A. S. ARGON, *Phil. Mag.* **28** (1973) 839.
20. E. J. KRAMER, private communication.
21. J. HARRIS, private communication.
22. M. P. STALLYBRASS, *Int. J. Eng. Sci.* **8** (1970) 351.
23. N. BROWN, B. D. METZGER and Y. IMAI, *J. Polymer. Sci. Phys. Edn.* **16** (1978) 1085.
24. Y. SATO, *Kobunshi Kagaku* **23** (1966) 69.
25. P. FARAHANI, unpublished work (1979).
26. V. HUNTINGDON, unpublished work (1977).
27. N. J. MILLS and N. WALKER, *J. Mater. Sci.* **15** (1980) 1832.

Received 20 October and accepted 14 November 1980.